Electrical Circuits (2)

Lecture 6

Magnetically Coupled Circuits

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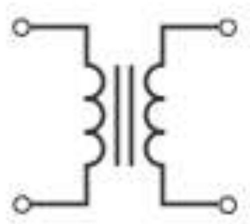
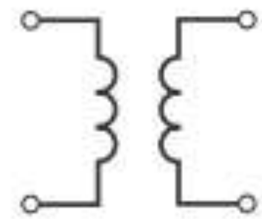
Application (Transformers)

Energy is transferred from the source to the load via the transformer’s magnetic field with no electrical connection between the two sides.



(a) Iron core



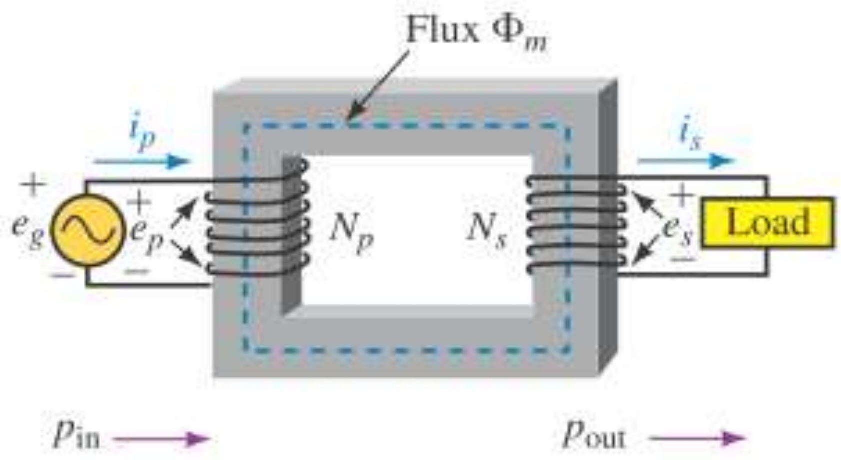
(b) Air core

Iron-Core Transformers: The Ideal Model

Air-Core Transformer: Loosely Coupled Model

Iron-Core Transformers: The Ideal Model

- Iron Core : All flux is confined to the core and links both windings. This is a "tightly coupled" transformer.
- It is described as Ideal: No power Loss



Voltage Ratio

$$e_p = N_p \frac{d\Phi_m}{dt} \qquad e_s = N_s \frac{d\Phi_m}{dt}$$

Re-arrange rate of change of flux in one side in both equations:

$$\frac{e_p}{e_s} = \frac{N_p}{N_s}$$

This ratio is called the turns ratio (or transformation ratio) and is given the symbol a.

$$a = N_p/N_s$$

Step-Up and Step-Down Transformers

- A step-up transformer is one in which the secondary voltage is higher than the primary voltage, **(a < 1)**
- A step-down transformer is one in which the secondary voltage is lower. **(a > 1)**

Current Ratio

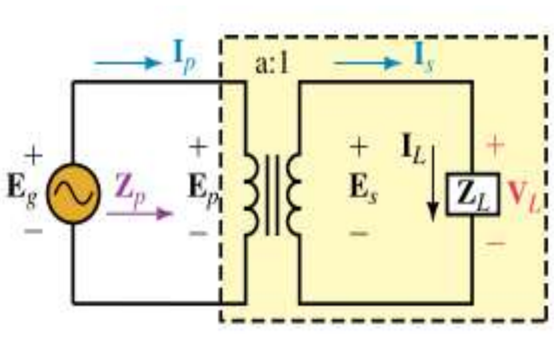
- Because an ideal transformer has no power loss, its efficiency is 100% and thus power in equals power out.

$$e_p i_p = e_s i_s \qquad \frac{i_p}{i_s} = \frac{e_s}{e_p} = \frac{1}{a}$$

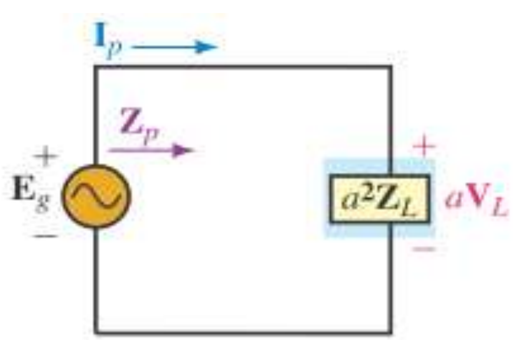
Reflected Impedance of Iron-core Transformer

- A transformer makes a load impedance look larger or smaller, depending on its turns ratio.

- When connected directly to the source, the load looks like impedance ZL , but when connected through a transformer, it looks like a^2 ZL .



(a) Actual circuit



(b) Reflected impedance $Z_p = a^2 Z_L$

$$Z_p = \frac{E_p}{I_p} = \frac{aE_s}{\left(\frac{I_s}{a}\right)} = a^2\frac{E_s}{I_s} = a^2\frac{V_L}{I_L}$$

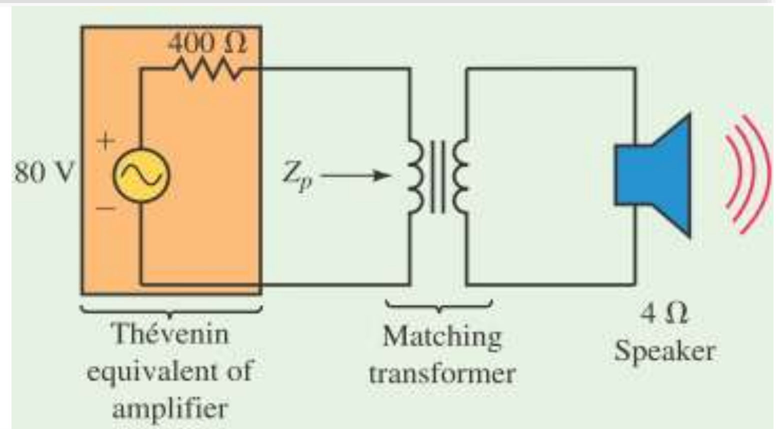
However $V_L/I_L = Z_L$. Thus,

$$Z_p = a^2 Z_L$$

Impedance Matching

- A transformer can be used to raise or lower the apparent impedance of a load by choice of turns ratio.
- This is referred to as impedance matching.
- Impedance matching is sometimes used to match loads to amplifiers to achieve maximum power transfer.

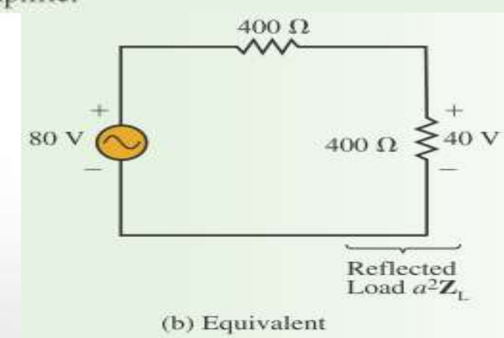
Example: If the speaker of Figure 23–29(a) has a resistance of 4 ohm, what transformer ratio should be chosen for max power? What is the power to the speaker?



Make the reflected resistance of the speaker equal to the internal (Thévenin) resistance of the amplifier.

$$Z_p = 400\ \Omega = a^2 Z_L = a^2(4\ \Omega).$$

$$a = \sqrt{\frac{Z_p}{Z_L}} = \sqrt{\frac{400\ \Omega}{4\ \Omega}} = \sqrt{100} = 10$$



Since half the source voltage appears across it.

power to Z_p is $(40\ V)^2/(400\ \Omega) = 4$ W.

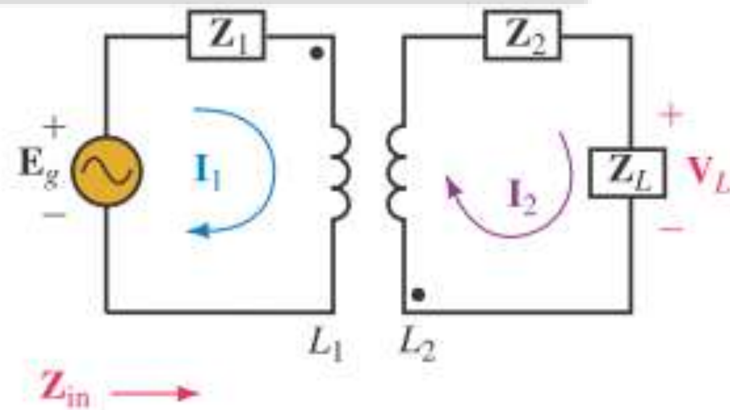
Reflected Impedance in Loosely Coupled Circuits

- Coupled circuits that do not have iron cores are said to be a loosely coupled.
- Those circuits cannot be characterized by turns ratios; rather, they are characterized by self and mutual inductances.
- Air-core transformers and general inductive circuit coupling fall into this category.

The impedance that you see reflected to the primary side from the secondary side is referred to as **coupled impedance**.

Loop 1: $\mathbf{E}_g - \mathbf{Z}_1\mathbf{I}_1 - j\omega L_1\mathbf{I}_1 - j\omega M\mathbf{I}_2 = 0$

Loop 2: $-j\omega L_2\mathbf{I}_2 - j\omega M\mathbf{I}_1 - \mathbf{Z}_2\mathbf{I}_2 - \mathbf{Z}_L\mathbf{L}_2 = 0$



which reduces to

$$\mathbf{E}_g = \mathbf{Z}_p\mathbf{I}_1 + j\omega M\mathbf{I}_2$$

$$0 = j\omega M\mathbf{I}_1 + (\mathbf{Z}_s + \mathbf{Z}_L)\mathbf{I}_2$$

$$\mathbf{Z}_p = \mathbf{Z}_1 + j\omega L_1$$

$$\mathbf{Z}_s = \mathbf{Z}_2 + j\omega L_2$$

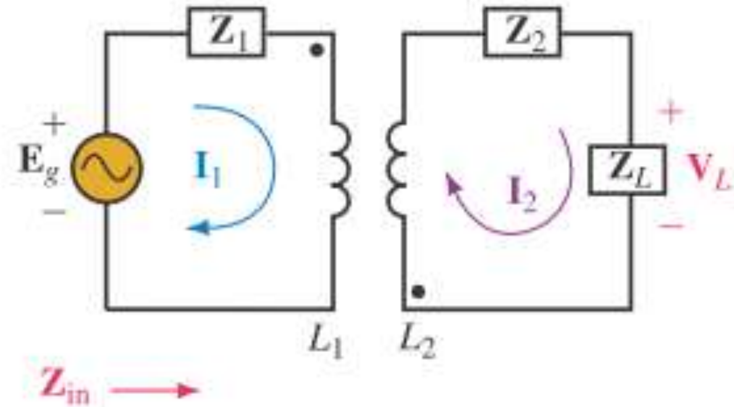
- Solving 2nd Equation for I2 and substituting this into 1st Equation yields, after some manipulation:

$$\mathbf{E}_g = \mathbf{Z}_p\mathbf{I}_1 + \frac{(\omega M)^2}{\mathbf{Z}_s + \mathbf{Z}_L}\mathbf{I}_1$$

Reflected Impedance in Loosely Coupled Circuits

- Now, divide both sides by I1 , and define

$$\mathbf{Z}_{in} = \mathbf{E}_g/\mathbf{I}_1.$$



$$\mathbf{E}_g = \mathbf{Z}_p\mathbf{I}_1 + \frac{(\omega M)^2}{\mathbf{Z}_s + \mathbf{Z}_L}\mathbf{I}_1$$

$$\mathbf{Z}_{in} = \mathbf{Z}_p + \frac{(\omega M)^2}{\mathbf{Z}_s + \mathbf{Z}_L}$$

$\frac{(\omega M)^2}{\mathbf{Z}_s + \mathbf{Z}_L}$ which reflects the secondary impedances into the primary, is the coupled impedance for the circuit.

- ✓ Note that since secondary impedances appear in the denominator, they reflect into the primary with reversed reactive parts.
- ✓ Thus, capacitance in the secondary circuit looks inductive to the source and vice versa for inductance